GIP 2024 Annual Meeting

University of Siegen

25–27 September 2024

Conference schedule

Time	Title
	Wednesday, 25.09.2024
09:45–10:00	**Opening Ceremony**
10:00–10:45	**Simon Hubmer** Frame decompositions in inverse problems and tomography
10:45–11:15	*Coffee Break*
11:15–12:00	**Christian Gerhards** Inverse problems in geomagnetism
12:00–13:15	*Lunch Break*
13:15–13:45	**Barbara Kaltenbacher** Ultrasound nonlinearity imaging in frequency domain
13:45–14:15	**Tobias Wolf** Nested Bregman iterations for decomposition problems
14:15–14:45	*Coffee Break*
14:45–15:15	**Tim Jahn** Early stopping of untrained convolutional neural networks
15:15–15:45	**Michael Quellmalz** Generalized Fourier diffraction theorem and filtered backpropagation for tomographic reconstruction
15:45–16:15	*Coffee Break*
16:15–16:45	**Milad Karimi** A uniqueness result of linearized holographic x-ray imaging from intensity correlations
16:45–17:15	**Sarah Eberle-Blick** The linearized monotonicity method for shape reconstruction based on the elasto-oscillatory wave equation
17:15–17:45	**Volker Michel** Learning a best basis for regularizing ill-posed inverse problems on the sphere or the ball – a status report
18:00–22:00	*Dinner in Freudenberg*
	Thursday, 26.09.2024
09:30–10:15	**Ruming Zhang** Inverse scattering problems in periodic waveguides
10:15–10:45	*Coffee Break*
10:45–11:15	**Thorsten Hohage** A unified variational analysis of non-standard noise models
11:15–11:45	**Philipp Mickan** Stability analysis for inverse random source problems
11:45–13:00	*Lunch Break*
13:00–13:45	**Todd Quinto** Alfred Louis: His Mathematics and Menschlichkeit
13:45–14:15	**Thomas Schuster** The method of approximate inverse: Alfred Louis' legacy
14:15–14:45	**Ming Jiang** Why CT is not working well for BMD

14:45–15:15	*Coffee Break*
15:15–15:45	**Andreas Rieder** On Kirchhoff migration formulas in 2D seismic imaging – microlocal analysis and AKL's approximate inverse
15:45–16:15	**Peter Maaß** Alfred Louis's lecture on special functions
16:15–16:45	**Ronny Ramlau** Big data inverse problems for the extremely large telescope
16:45–17:15	*Coffee Break*
17:15–17:45	**Marco Pauleti** Inexact Newton regularizations with uniformly convex stability terms
17:45–18:15	**Simon Hackl** Ultrasound aberration correction for layered media
Friday, 27.09.2024	
09:30–10:15	**Anne Wald** Inverse problems related to biophysical experiments
10:15–10:45	*Coffee Break*
10:45–11:00	**GIP–PhD prize ceremony award**
11:00–11:45	**Scientific talk of PhD prize awardee**
11:45–12:45	*Lunch Break*
12:45–13:15	**Kathrin Hellmuth** Identifiability analysis for PDE parameter reconstruction
13:15–13:45	**Oleh Melnyk** Time-harmonic optical flow with applications in elastography
13:45–14:15	**Tim-Jonas Peter** Reconstructing wind fields from gravitational data on gas giants – an investigation of mathematical methods
14:15–14:30	**Closing Ceremony**

List of abstracts

PD Dr. Sarah Eberle-Blick University of Frankfurt

The linearized monotonicity method for shape reconstruction based on the elasto-oscillatory wave equation

We solve the inverse problem of the elasto-oscillatory wave equation by means of monotonicity methods. In more detail, we start with the standard monotonicity method for shape reconstruction of inclusions in elastic bodies by using time-harmonic elastic waves. In order to accelerate the standard monotonicity method, we derive a linearized version. The linearized method provides an efficient version of the method, drastically reducing computation time. In addition, we compare these methods with each other and present several numerical test examples.

Prof. Dr. Christian Gerhards TU Freiberg

Inverse problems in geomagnetism

A wide range of inverse problems arise in the context of geomagnetism and planetary magnetism, e.g., the separation of contributions of core, crustal, and external origin, the estimation of driving forces in the core, or the reconstruction of underlying magnetizations. We focus on the rather simple setup of inverse magnetization problems in a potential field context. This plays a role for paleomagnetism at planetary as well as microscopic scale and for applied geophysics in exploration. Problems arise due to an intrinsic non-uniqueness, instability, as well as often only locally/regionally available data. A priori known localization of the sources can improve on some of these issues. This talk gives an overview on the modeling of such problems and corresponding results.

DI Simon Hackl Johannes Kepler University Linz

Ultrasound aberration correction for layered media

Focused ultrasound is an important tool for non-invasive diagnostic and therapeutic methods and therefore it is widely used in modern medicine. A crucial assumption in medical applications is a constant sound speed in the observed medium. Non-constant sound speeds in real media lead to deteriorated image quality. Although it is possible to incorporate a space dependent sound speed into the model, this comes at the cost of significantly higher computational complexity, which disqualifies this approach for the clinical practice. Therefore, a significantly faster method is needed. In this talk, we present an adapted ultrasound focusing method based on geometrical acoustics. It is assumed that the observed medium consists of layers with known constant sound speeds, seperated by known C^1 medium boundaries. In this setting, we can model the aberrations caused by a layered medium and derive adapted focusing algorithms that correct for them. Finally, we conduct numerical simulations to validate our derived methods.

Kathrin Hellmuth University of Würzburg

Identifiability analysis for PDE parameter reconstruction

Identifiability analysis describes a structured framework to study the influence of different sources of non-identifiability, i.e. ill-posedness, on the inverse problem and is connected. Different remedies are required for different sources. For instance, an intrinsic (structural) non-identifiability requires fundamental changes in the model or the experimental measurements, whereas practical non-identifiability can be cured by improved experimental design, reduction of noise in the measurements or Tikhonov regularization. In this sense, identifiability analysis is connected to modelling, experimental planning and design as well as numerical reconstruction practices. Being developed from control theory, techniques have ripened in the ODE model setting and are widely adopted e.g. in systems biology modelling and model fitting. A direct translation to the PDE context is not always possible and up to today, this framework has only begun to be used for PDEs inverse problems. We are convinced that the structure it provides is beneficial in the PDE context as well and can help classify obtained results and shed light on further questions. This shall be illustrated by an example from a parameter reconstruction problem for the kinetic chemotaxis equation. This is based on joint work with Christian Klingenberg (JMU Würzburg), Qin Li (UW Madison-Wisconsin) and Min Tang (Shanghai Jiao-Tong).

Prof. Dr. Thorsten Hohage University of Göttingen

A unified variational analysis of non-standard noise models

In much of the literature on regularization theory of inverse problems, errors in the data are simplistically modeled by an (infinite-dimensional) vector in the image space of the forward operator or by (Gaussian) white noise. Papers on more realistic noise models often consider rather special settings concerning the forward operator and/or the smoothness of the exact solution. In this talk we present a unified framework for the analysis of general, more realistic noise models in variational regularization. The central assumption is called a variational noise bound, and it naturally complements the well-studied variational source conditions describing the smoothness of the exact solution. We show that our assumptions imply existence of minimizers, the proof of which can be a tedious task in case of large noise where the data term is not bounded from below.

We show that our setting can be used to analyze discrete data, Gaussian white noise, Poisson noise, errors in the operator, and impulsive noise. In cases where lower bounds are known, our error bounds are of optimal order. In several cases previous proofs are considerably simplified.

(Joint work with Frank Werner)

Dr. Simon Hubmer Johannes Kepler University Linz

Frame decompositions in inverse problems and tomography

In this talk, we consider the solution of linear inverse problems and their regularization via frame decompositions (FDs), which are generalizations of the singular-value decomposition (SVD). Similarly to the SVD, these FDs encode information on the structure and ill-posedness of a given problem, and can be used as the basis for the design and implementation of efficient numerical solution methods. We show that in contrast to the SVD, FDs can be derived explicitly for a wide class of operators. This in particular includes operators satisfying certain stability conditions such as the Radon or the Funk-Radon transform. We then consider various theoretical aspects of FDs such as recipes for their construction and solution properties of the reconstruction formulas induced by them. Furthermore, we discuss convergence and convergence rates results for continuous regularization methods based on FDs under both a-priori and a-posteriori parameter choice rules. Finally, we consider the practical utility of FDs for solving inverse problems by considering numerical examples from atmospheric and computerized tomography.

Dr. Tim Jahn TU Berlin

Early stopping of untrained convolutional neural networks

In recent years, new regularization methods based on (deep) neural networks have shown very promising empirical performance for the numerical solution of ill-posed problems, such as in medical imaging and imaging science. Due to the nonlinearity of neural networks, these methods often lack satisfactory theoretical justification. In this work, we rigorously discuss the convergence of a successful unsupervised approach that utilizes untrained convolutional neural networks to represent solutions to linear ill-posed problems. Untrained neural networks have particular appeal for many applications because they do not require paired training data. The regularization property of the approach relies solely on the architecture of the neural network instead. Due to the vast over-parameterization of the employed neural network, suitable early stopping is essential for the success of the method. We establish that the classical discrepancy principle is an adequate method for early stopping of two-layer untrained convolutional neural networks learned by gradient descent, and furthermore, it yields an approximation with minimax optimal convergence rates.

Prof. Dr. Barbara Kaltenbacher University of Klagenfurt

Ultrasound nonlinearity imaging in frequency domain

In this talk we discuss the problem of imaging of the nonlinearity coefficient in ultrasonics. We show derivation of a multiharmonic system that allows to model the forward problem in frequency domain and present a linearized uniqueness proof.

Milad Karimi University of Göttingen

A uniqueness result of linearized holographic x-ray imaging from intensity correlations

Holographic coherent X-ray imaging enables nanoscale imaging of biological cells and tissue, rendering both phase and absorption contrast, i.e. real and imaginary parts of the refractive index. A primary challenge of this imaging technique is radiation damage. We present a different modality of this imaging technique using a partially coherent incident beam and time-resolved intensity measurements based on new measurement technologies. This enables the acquisition of intensity correlations in addition to the commonly used expectations of intensities. In this paper, we explore the information content of these intensity correlations, demonstrating analytically that both phase and absorption contrast can be uniquely determined in a linearized model by the intensity correlation data. The uniqueness theorem is established by reducibilty theory of positive semi-definite kernels and symmetry properties between the perturbation and the kernels in Fourier space. For regularized reconstructions, it is important to take into account the statistical distribution of the correlation data. The measured intensity data are, in principle, described by Cox-processes, roughly speaking a Poisson process with random intensity. A big challenge in practice is the increase of dimensionality when we computing the correlations from intensity data in a preprocessing stage. A novel approach to address this challenge is proposed. Basically, it applies to intensity data while implicitly using the full information content of the correlation data, providing quantitative estimates and convergence by iterations. Finally, the potential of this approach confirms *all-at-once* reconstructions of phase and absorption contrasts from only intensity correlations, eliminating the need for mean intensities.

Dr. Marvin Knöller Karlsruhe Institute of Technology

Maximizing the electromagnetic chirality of thin metallic nanowires in the visible spectrum

Electromagnetic chirality (em-chirality) describes differences in the interaction of scatterers, materials or metamaterials with fields of opposite helicities. An object is said to be maximally em-chiral if it does not interact with fields of one helicity at all, meaning that such an object would be invisible with respect to fields of one helicity. For individual scatterers, em-chirality can be quantified by chirality measures that map the far field operator of the scatterer to a value between zero (no chiral interaction) and one (maximal em-chirality). It has been observed that metallic helices exhibit strong em-chiral effects for frequencies in the infrared spectrum. However, towards the optical regime, these chiral effects become weaker. In this talk we focus on the design of highly em-chiral thin metallic nanowires at optical frequencies. We present a gradient-based optimization scheme that maximizes em-chirality with respect to the center curve of the nanowire and with respect to the rotation of the nanowire's cross section around the center curve. This scheme is based on an asymptotic representation formula for thin tubular scatterers. We show a series of numerical examples highlighting the optimized nanowires. At optical frequencies we find that these objects exceed the chirality measures obtained by classical metallic helices.

Dr. Oleh Melnyk TU Berlin

Time-harmonic optical flow with applications in elastography

In this talk, we propose mathematical models for reconstructing the optical flow in time-harmonic elastography. In this image acquisition technique, the object undergoes a special time-harmonic oscillation with known frequency so that only the spatially varying amplitude of the velocity field has to be determined. This allows for a simpler multi-frame optical flow analysis using Fourier analytic tools in time. We propose three variational optical flow models and show how their minimization can be tackled via Fourier transform in time. Numerical examples with synthetic as well as real-world data demonstrate the benefits of our approach.

Prof. Dr. Volker Michel University of Siegen

Learning a best basis for regularizing ill-posed inverse problems on the sphere or the ball – a status report

Ill-posed inverse problems on spheres and balls are significant in Earth sciences and medical imaging. Over the decades, a cornucopia of trial functions has become available for expanding approximate solutions. They range from global functions (orthogonal polynomials) to localized functions (such as scaling functions and wavelets in terms of radial basis functions). While each basis system has its peculiar pros and cons, a diverse set of trial functions promises synergies. For instance, global trends in the solution might be covered best with orthogonal polynomials, while scaling functions might suit well for adding regional structures and wavelets could be even more adequate to refine the solution by some local details. The Geomathematics Group Siegen has developed several versions of a matching pursuit type algorithm (sometimes summarized as IPMPs - inverse problems matching pursuits), which try to construct such a diverse basis system. The starting point is the choice of a very large and redundant set of trial functions (known as the dictionary) and the objective of minimizing a Tikhonov-Phillips functional (tailored to the specific inverse problem and a Sobolev norm on the sphere or the ball). More recently, we accomplished the use of an infinite dictionary. Note that radial basis functions have continuous parameters (in contrast e.g. to the discrete degrees and orders of spherical harmonics basis functions). Thus, all parameters within a given interval range are now available and no bias might be produced by discretizing this range. Due to some analogy to dictionary learning approaches, we call these variants the Learning IPMPs (LIPMPs). So far, the IPMPs have been successfully applied to gravity field modelling and inversion and to seismic tomography in Earth sciences as well as to EEG-MEG-data inversion in medical imaging. In this talk, the current status of the development is summarized with a focus on benefits and remaining shortcomings.

Phillip Mickan University of Göttingen

Stability analysis for inverse random source problems

The problem under investigation is to determine the strength of a random acoustic source from correlations of measurements distant from the source region. Specifically, we acquire measurements of the time-harmonic acoustic waves on a surface surrounding the source region and then average their correlation to approximate the covariance operator of the solution process on the measurement surface. A natural extension of the existing uniqueness results [1,4] are stability estimates. We were able to show in general settings that this problem is severely ill-posed. Particularly, we show two bounds the upper bound usually referred to as stability estimate and the lower bound called instability estimate. The stability estimate is shown by verifying a variational source condition [3] for the problem which in turn also provides convergence rates for a variety of spectral regularisation methods. Instability is based on a general entropy argument presented for operators of the type $X \to \mathcal{L}(H, H')$ with X some metric space and H some separable Hilbert space [2]. The talk will be finished with some numerical experiments that support our theoretical results.

References

[1] A.J. Devaney. The inverse problem for random sources. In *Journal of Mathematical Physics* 20.8 (1979), pp. 1687-1691.
[2] M. Di Cristo and L. Rondi. Examples of exponential instability for inverse inclusion and scattering problems. In *Inverse Problems*, 19 (2003), p. 685.
[3] T. Hohage and F. Weidling. Characterizations of variational source conditions, converse results, and maxisets of spectral regularization methods. In *SIAM* Journal on Numerical Analysis, 55.2 (2017), pp. 698-630.
[4] T. Hohage, H.-G. Raumer, and C. Spehr. Uniqueness of an inverse source problem in experimental aeroacoustics. In *Inverse Problems* 36(7) (2020).

Marco Pauleti Karlsruhe Institute of Technology

Inexact Newton regularizations with uniformly convex stability terms: A unified convergence analysis

In this talk, we present a unified convergence analysis of inexact Newton regularizations with general uniformly convex penalty terms for nonlinear ill-posed problems in Banach spaces. These schemes consist of an outer (Newton) iteration and an inner iteration, which provides the update of the current outer iterate. To this end, the nonlinear problem is linearized about the current iterate and the resulting linear system is approximately (inexactly) solved by an inner regularization method. In our analysis, we only rely on generic assumptions of the inner methods, and we show that a variety of regularization techniques satisfies these assumptions. For instance, gradient-type and iterated-Tikhonov methods are covered. Numerical experiments based on the inverse problem of electrical impedance tomography illustrate the impact of different uniformly convex penalty terms.

Dr. Michael Quellmalz TU Berlin

Generalized Fourier diffraction theorem and filtered backpropagation for tomographic reconstruction

We consider diffraction-tomographic reconstruction of an object characterized by its scattering potential. We establish a rigorous generalization of the Fourier diffraction theorem in arbitrary dimension, giving a precise relation in the Fourier domain between measurements of the scattered wave and reconstructions of the scattering potential. With this theorem at hand, Fourier coverages for different experimental setups are investigated taking into account parameters such as object orientation, direction of incidence and frequency of illumination. Allowing for simultaneous and discontinuous variation of these parameters, a general filtered backpropagation formula is derived resulting in an explicit approximation of the scattering potential for a large class of experimental setups.

Prof. Dr. Andreas Rieder Karlsruhe Institute of Technology

On Kirchhoff migration formulas in 2D seismic imaging: microlocal analysis and AKL's approximate inverse

The term "Kirchhoff migration" refers to a collection of approximate linearized inversion formulas for solving the inverse problem of seismic tomog-

raphy, which entails reconstructing the Earth's subsurface from reflected wave fields. There are a number of such formulas, the first of which date back to the 1950s. In this talk we systematically compare old and new formulas in 2D from a microlocal point of view. To this end, we consider the corresponding imaging operators in a unified framework as pseudodifferential or Fourier integral operators. Numerical examples based on Alfred Louis' concept of approximate inverse illustrate the theoretical insights and allow a visual comparison of the different formulas.

Prof. Dr. Thomas Schuster Saarland University

The method of approximate inverse: Alfred Louis' legacy

The method of approximate inverse goes back to the seminal article of Maaß and Louis from 1990. It represents a class of regularization methods for operator equations of first kind which relies on the computation of a stable solution approximation by evaluating inner products of the given data and reconstruction kernels. These kernels in turn are associated with the adjoint of the forward operator and a mollifier, i.e., a smooth approximation of the delta distribution. The talk outlines the history of this method starting with the article from 1990, summarizing the main theoretical results and application fields and ends with an outlook to future research. Alfred Louis was a pioneer of this method. He developed, analyzed and applied this method in various fields.

JProf. Dr. Anne Wald University of Göttingen

Inverse problems related to biophysical experiments

Many processes in cells are driven by the interaction of multiple proteins, for example cell contraction, division or migration. The shape of a cell and its dynamics is largely determined by the cytoskeleton, a network of various protein filaments. Myosin motor proteins are able to bind to and move along filaments by converting energy into motion, which creates dynamic networks and yields the basis for cell deformation. A major goal in biophysics is to gain information on the mechanical properties of cells or, in general, active matter. One key quantity are the forces that are generated due to the activity. In this talk, we take a look at two inverse problems arising in biophysics, where such forces are to be determined from indirect measurements. One is traction force microscopy, a popular method to reconstruct traction forces exterted by a cell on an elastic substrate. A further experiment aims at finding the forces driving fluid flow inside an actomyosin droplet, which is a model system for active matter. We will take a look at the underlying physical models, the mathematical analysis of the inverse problems as well as numerical results for experimental data.

Tobias Wolf University of Klagenfurt

Nested Bregman iterations for decomposition problems

We focus on solving linear inverse problems, where the solution is a sum of components with different properties. A commonly used method for this is to use variational methods with an infimal convolution of regularizing functions. While for noise-corrupted data, good approximations of the true solution can be obtained by Bregman iterations, the quality of the single components depends on the proper choice of weights associated with the infimally convoluted functions. In order to overcome the weighting choice, we propose the method of Nested Bregman iterations to improve a decomposition iteratively. We discuss the convergence behavior and well-definedness of the proposed method, and illustrate its strength numerically for various examples.

Prof. Dr. Ruming Zhang TU Berlin

Inverse scattering problems in periodic waveguides

Inverse scattering problems in periodic waveguides are central to many areas of applied mathematics, physics, and engineering, particularly in the design and analysis of photonic and phononic devices. These problems involve determining the properties of a medium or structure from scattered wave data, embedded in periodic waveguides. In this talk, we begin with the discussion of the mathematical framework, focusing on the Helmholtz equation in periodic media and the role of Floquet-Bloch theory in simplifying the problem. We will also introduce recent developments in the analysis an efficient numerical solvers for the direct scattering problems. Next, we will review important methods and results for the inverse scattering problems in recent decades. The methods will involve the optimisation methods, linear sampling methods, and monotonicity methods.